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LETTER TO THE EDITOR

Raising operators for the osp(1/2N, $\mathbb{R}$) orthosymplectic Lie superalgebras

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Abstract. The branching rule is obtained for the decomposition of the osp(1/2N, $\mathbb{R}$) oscillator-like unitary irreducible representations into sp(2N, $\mathbb{R}$) representations. Raising operators for osp(1/2N, $\mathbb{R}$) $\supset$ sp(2N, $\mathbb{R}$) are determined and used to construct a basis in the irreducible representation carrier space. The matrix elements of the odd generators between two sp(2N, $\mathbb{R}$) lowest-weight states are calculated. The general results so obtained are illustrated with the osp(1/4, $\mathbb{R}$) example.

Shift operators are known to be very useful for constructing a basis in the carrier space of a Lie group unitary irreducible representation (irrep). In particular, whenever the carrier space contains a highest- (lowest-) weight state, the whole space may be obtained by applying appropriate lowering (raising) operators to the highest- (lowest-) weight state. Shift operators were explicitly constructed for all the classical compact groups, for many non-compact groups, and for some non-semisimple groups (Nagel and Moshinsky 1965, Pang and Hecht 1967, Wong 1967, 1974, Patera 1973, Patera *et al* 1974, Hughes 1973, Hughes and Yadegar 1978, Bincer 1977, 1978a, b, 1982, Quesne 1987, 1988).

During recent years, the increasing importance of supersymmetry concepts in physics has stimulated the study of the representation theory of Lie superalgebras and supergroups (Scheunert 1979, Bars 1985). In such a context, shift operators also prove to be very useful, although up to now full advantage has not been taken of them. Their usefulness was illustrated by Hughes (1981), who employed them to classify the irreps of the osp(1/2) and osp(1/2, $\mathbb{R}$) orthosymplectic superalgebras, whose even part is the Lie algebra usp(2) or sp(2, $\mathbb{R}$).

In this letter, we construct raising operators for the oscillator-like unitary irreps (Bars and Günaydin 1983) of the osp(1/2N, $\mathbb{R}$) orthosympletic superalgebras, thereby providing a basis of their carrier space, classified according to the chain osp(1/2N, $\mathbb{R}$) $\supset$ sp(2N, $\mathbb{R}$) $\supset$ u(N) $\supset$ u($N-1$) $\supset$ ... $\supset$ u(1). This is the first step in the resolution of a similar problem for the class of superalgebras osp(M/2N, $\mathbb{R}$), when reduced to their subalgebra so(M)$\oplus$sp(2N, $\mathbb{R}$). Such superalgebras play an important role in applications, among others because the corresponding supergroups OSP(M/2N,$\mathbb{R}$) are the groups of canonical transformations for mixed systems of bosons and fermions (de Crombrugghe and Rittenberg 1983, Balantekin *et al* 1988). The orthosymplectic superalgebras with $M = 1$ are the simplest to deal with, since they are the only superalgebras

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for which the direct products of all finite-dimensional representations are completely reducible (Djoković and Hochschild 1976). Whenever $M > 1$, one needs to treat typical and atypical representations individually.

A basis for the even part of $\mathrm{osp}(1/2N, \mathbb{R})$ consists of the $\mathrm{sp}(2N, \mathbb{R})$ generators $E_{ij} = (E_{ji})^\dagger$, $D^\dagger_{ij}$, $D_{ij} = (D^\dagger_{ij})^\dagger$, $i, j = 1, \ldots, N$, and the odd part has basis elements F_i, G_i, $i = 1, \ldots, N$, which form a $2N$-dimensional vector representation of the $\mathrm{sp}(2N, \mathbb{R})$ algebra. The non-zero commutation and anticommutation relations satisfied by these elements are

$$\begin{aligned}
&[E_{ij}, E_{kl}] = \delta_{jk}E_{il} - \delta_{il}E_{kj} \\
&[E_{ij}, D^\dagger_{kl}] = \delta_{jk}D^\dagger_{il} + \delta_{jl}D^\dagger_{ik} \qquad [E_{ij}, D_{kl}] = -\delta_{ik}D_{jl} - \delta_{il}D_{jk} \\
&[D_{ij}, D^\dagger_{kl}] = \delta_{ik}E_{lj} + \delta_{il}E_{kj} + \delta_{jk}E_{li} + \delta_{jl}E_{ki} \\
&[E_{ij}, F_k] = -\delta_{ik}F_j \qquad [E_{ij}, G_k] = \delta_{jk}G_i \\
&[D^\dagger_{ij}, F_k] = -\delta_{ik}G_j - \delta_{jk}G_i \qquad [D_{ij}, G_k] = \delta_{ik}F_j + \delta_{jk}F_i \\
&\{F_i, F_j\} = D_{ij} \qquad \{F_i, G_j\} = E_{ji} \qquad \{G_i, G_j\} = D^\dagger_{ij}.
\end{aligned} \tag{1}$$

Here E_{ii}, $i = 1, \ldots, N$, are the $\mathrm{sp}(2N, \mathbb{R})$ weight generators, while E_{ij} $(i<j)$, $D^\dagger_{ij}$, G_i, and E_{ij} $(i>j)$, D_{ij}, F_i respectively raise or lower the $\mathrm{sp}(2N, \mathbb{R})$ weight. Note that E_{ij}, $i, j = 1, \ldots, N$, span the $u(N)$ subalgebra of $\mathrm{sp}(2N, \mathbb{R})$.

It is straightforward to show that no gradestar representations (Scheunert *et al* 1977) of $\mathrm{osp}(1/2N, \mathbb{R})$ can occur, but that there are two classes of star representations, depending on whether $G_i = F^\dagger_i$ or $G_i = -F^\dagger_i$. In the present work, we shall restrict ourselves to oscillator-like unitary irreps in a super Fock space (Bars and Günaydin 1983), in which case $G_i = F^\dagger_i$. This super Fock space $\mathscr{F}$ is defined as the tensor product of the Fock spaces of Nn pairs of boson creation and annihilation operators $b^\dagger_{is}$, b_{is}, $i = 1, \ldots, N$, $s = 1, \ldots, n$, and n pairs of fermion operators $a^\dagger_s$, a_s, $s = 1, \ldots, n$. In $\mathscr{F}$, the $\mathrm{osp}(1/2N, \mathbb{R})$ generators can be written as

$$E_{ij} = \sum_{s=1}^{n} b^\dagger_{is}b_{js} + \tfrac{1}{2}n\delta_{ij} \qquad D^\dagger_{ij} = \sum_{s=1}^{n} b^\dagger_{is}b^\dagger_{js} \qquad D_{ij} = \sum_{s=1}^{n} b_{is}b_{js} \tag{2a}$$

$$F^\dagger_i = 2^{-1/2} \sum_{s=1}^{n} b^\dagger_{is}(a^\dagger_s + a_s) \qquad F_i = 2^{-1/2} \sum_{s=1}^{n} b_{is}(a^\dagger_s + a_s). \tag{2b}$$

In the realisation (2a), the $\mathrm{sp}(2N, \mathbb{R})$ algebra has only positive discrete series irreps, the so-called harmonic series irreps (King and Wybourne 1985). They are characterised by their lowest-weight $\boldsymbol{\omega}$, and denoted by $\langle\boldsymbol{\omega}\rangle$. Here $\boldsymbol{\omega}$ is a shorthand notation for $\omega_1\omega_2 \ldots \omega_N$, and ω_i is given by $\omega_i = \lambda_i + n/2$, where $[\lambda_1\lambda_2 \ldots \lambda_N]$ may be any partition into non-negative integers (provided that $n \geqslant 2N$, which we shall henceforth assume). The lowest-weight state (LWS) $|\boldsymbol{\omega}\rangle$ of $\langle\boldsymbol{\omega}\rangle$ satisfies the equations (Deenen and Quesne 1984)

$$E_{ii}|\boldsymbol{\omega}\rangle = \omega_{N+1-i}|\boldsymbol{\omega}\rangle \tag{3a}$$

$$E_{ij}|\boldsymbol{\omega}\rangle = 0 \qquad i>j \tag{3b}$$

$$D_{ij}|\boldsymbol{\omega}\rangle = 0. \tag{3c}$$

Note that some additional quantum numbers, which are omitted, may be necessary to completely specify $|\boldsymbol{\omega}\rangle$.

In a given osp$(1/2N, \mathbb{R})$ irrep, let us select an sp$(2N, \mathbb{R})$ LWS $|\boldsymbol{\omega}\rangle$. If such a state is not annihilated by F_1, then we replace it by $F_1|\boldsymbol{\omega}\rangle$. As it results from (1) and (3), the latter is annihilated by F_1, while satisfying equations similar to (3), thence being the LWS of an sp$(2N, \mathbb{R})$ irrep. Having selected an sp$(2N, \mathbb{R})$ LWS annihilated by F_1, we then proceed in a similar way with F_2 and construct an sp$(2N, \mathbb{R})$ LWS annihilated by both F_1 and F_2. Having considered the remaining F_i, $i = 3, \ldots, N$, we finally arrive at a state $|\boldsymbol{\Omega}\rangle$, satisfying the equations

$$E_{ii}|\boldsymbol{\Omega}\rangle = \Omega_{N+1-i}|\boldsymbol{\Omega}\rangle \qquad E_{ij}|\boldsymbol{\Omega}\rangle = 0\ (i>j) \qquad D_{ij}|\boldsymbol{\Omega}\rangle = 0 \qquad F_i|\boldsymbol{\Omega}\rangle = 0. \tag{4}$$

From (4), it follows that $|\boldsymbol{\Omega}\rangle$ is the LWS of the lowest-weight sp$(2N, \mathbb{R})$ irrep $\langle\boldsymbol{\Omega}\rangle$, contained in the considered osp$(1/2N, \mathbb{R})$ irrep. Hence, the latter has a lowest weight, which may serve to characterise it. In order to distinguish it from the sp$(2N, \mathbb{R})$ irrep $\langle\boldsymbol{\Omega}\rangle$, built on the same LWS, we shall use the mixed orthogonal-symplectic notation $(\boldsymbol{\Omega}\rangle$ for the osp$(1/2N, \mathbb{R})$ irrep.

All the states belonging to the carrier space of $(\boldsymbol{\Omega}\rangle$ may now be obtained from $|\boldsymbol{\Omega}\rangle$ by applying the elements of the osp$(1/2N, \mathbb{R})$ universal enveloping algebra. From the Poincaré–Birkhoff–Witt theorem, a basis of the latter is made of the operators

$$\left(\prod_{i\leqslant j}(D_{ij}^\dagger)^{k_{ij}}\right)\left(\prod_i (F_i^\dagger)^{l_i}\right)\left(\prod_{i<j}(E_{ij})^{m_{ij}}\right)$$
$$\times\left(\prod_i (E_{ii})^{m_{ii}}\right)\left(\prod_{i>j}(E_{ij})^{m_{ij}}\right)\left(\prod_i (F_i)^{n_i}\right)\left(\prod_{i\leqslant j}(D_{ij})^{p_{ij}}\right) \tag{5}$$

where k_{ij}, m_{ij}, $p_{ij} \in \mathbb{N}$, l_i, $n_i \in \{0, 1\}$, and we used (1) to put all the raising (weight) generators on the left of the weight (lowering) generators. From (4), the action of (5) on $|\boldsymbol{\Omega}\rangle$ gives rise to the states

$$|\boldsymbol{\Omega}(\Omega)\boldsymbol{kl}\rangle = \left(\prod_{i\leqslant j}(D_{ij}^\dagger)^{k_{ij}}\right)\left(\prod_i (F_i^\dagger)^{l_i}\right)|\boldsymbol{\Omega}(\Omega)\rangle \tag{6}$$

where $k_{ij} \in \mathbb{N}$, $l_i \in \{0, 1\}$, $|\boldsymbol{\Omega}(\Omega)\rangle$ denotes a Gel'fand state of the $u(N)$ irrep $[\boldsymbol{\Omega}]$ containing the LWS $|\boldsymbol{\Omega}\rangle$, and (Ω) is a Gel'fand pattern (Gel'fand and Tseitlin 1950). The set of states (6) form a non-orthonormal basis of $(\boldsymbol{\Omega}\rangle$ which we shall call the monomial basis.

The monomial basis states are neither characterised by a definite sp$(2N, \mathbb{R})$ irrep, nor by a definite $u(N)$ irrep. A $u(N)$ basis can, however, be easily constructed. For this purpose, we note that $F_i^\dagger$, $i = 1, \ldots, N$, transform under the $u(N)$ irrep $[1]$, and that the product of two operators $F_i^\dagger$ and $F_j^\dagger$ can be written as

$$F_i^\dagger F_j^\dagger = \tfrac{1}{2}(D_{ij}^\dagger + [F_i^\dagger, F_j^\dagger]) \tag{7}$$

where $D_{ij}^\dagger$ and $[F_i^\dagger, F_j^\dagger]$ transform under the $u(N)$ irreps $[2]$ and $[1^2]$ respectively. Hence, $u(N)$ basis states can be written as

$$|\boldsymbol{\Omega}[1^l]\boldsymbol{\omega\nu\rho h}(h)\rangle = [P_{\boldsymbol{\nu}}(\boldsymbol{D}^\dagger) \times [Q_{[1^l]}(\boldsymbol{F}^\dagger) \times |\boldsymbol{\Omega}\rangle]^{\boldsymbol{\omega}}]^{\rho\boldsymbol{h}}_{(h)}. \tag{8}$$

In (8), $P_{\boldsymbol{\nu}}(\boldsymbol{D}^\dagger)$ is a polynomial in the $D_{ij}^\dagger$ operators, characterised by a given $u(N)$ irrep $[\boldsymbol{\nu}]$, where ν_i, $i = 1, \ldots, N$, are non-negative even integers (Deenen and Quesne 1982); $Q_{[1^l]}(\boldsymbol{F}^\dagger)$ is obtained by antisymmetrising a product of l operators $F_i^\dagger$ and transforms under the $u(N)$ irrep $[1^l]$; the square brackets denote couplings of $u(N)$ irreps, either that of $[\boldsymbol{\Omega}]$ and $[1^l]$ to $[\boldsymbol{\omega}]$, or that of $[\boldsymbol{\omega}]$ and $[\boldsymbol{\nu}]$ to $[\boldsymbol{h}]$; ρ distinguishes between repeated irreps $[\boldsymbol{h}]$, and (h) is a Gel'fand pattern of $[\boldsymbol{h}]$. When l runs over the set $\{0, 1, \ldots, N\}$, the labels $\omega_1, \ldots, \omega_N$ take all the values satisfying the inequalities

$$\Omega_1 \leqslant \omega_1 \leqslant \Omega_1 + 1 \qquad \Omega_i \leqslant \omega_i \leqslant \min(\Omega_i + 1, \omega_{i-1}) \qquad i = 2, \ldots, N. \tag{9}$$

If we list in order of increasing weight all the $u(N)$ irreps $[\boldsymbol{h}]$, which appear in (8) for a given $\boldsymbol{\Omega}$, we just obtain the $u(N)$ content of the set of sp$(2N, \mathbb{R})$ irreps $\langle\boldsymbol{\omega}\rangle$, whose labels fulfill the inequalities (9). Hence, the branching rule for the decomposition of the osp$(1/2N, \mathbb{R})$ irrep $(\boldsymbol{\Omega}\rangle$ into sp$(2N, \mathbb{R})$ irreps $\langle\boldsymbol{\omega}\rangle$ is given by

$$(\boldsymbol{\Omega}\rangle\downarrow \sum_{\omega_1=\Omega_1}^{\Omega_1+1} \sum_{\omega_2=\Omega_2}^{\min(\Omega_2+1,\,\omega_1)} \cdots \sum_{\omega_N=\Omega_N}^{\min(\Omega_N+1,\,\omega_{N-1})} \oplus\langle\boldsymbol{\omega}\rangle. \tag{10}$$

A generic osp$(1/2N, \mathbb{R})$ irrep therefore contains 2^N sp$(2N, \mathbb{R})$ irreps. This generalises the well known 'dispin' structure of the osp$(1/2, \mathbb{R})$ irreps (Hughes 1981).

Basis states of $(\boldsymbol{\Omega}\rangle$, classified according to the chain osp$(1/2N, \mathbb{R}) \supset$ sp$(2N, \mathbb{R}) \supset$ u$(N) \supset$ u$(N-1) \supset \ldots \supset$ u(1), may be written as

$$|\boldsymbol{\Omega\omega\nu}\rho\boldsymbol{h}(h)\rangle\rangle = [P_{\nu}(\boldsymbol{D}^{\dagger}) \times |\boldsymbol{\Omega\omega}\rangle]^{\rho\boldsymbol{h}}_{(h)} \tag{11}$$

where $|\boldsymbol{\Omega\omega}\rangle$ denotes the LWS of $\langle\boldsymbol{\omega}\rangle$ contained in $(\boldsymbol{\Omega}\rangle$, and there is a coupling of the $u(N)$ irreps $[\boldsymbol{\omega}]$ and $[\boldsymbol{\nu}]$ to $[\boldsymbol{h}]$. Except for $l = 0$ or 1, the states

$$[Q_{[1^l]}(\boldsymbol{F}^{\dagger}) \times |\boldsymbol{\Omega}\rangle]^{\boldsymbol{\omega}}_{(\min)}$$

where (min) is the lowest-weight Gel'fand pattern of $[\boldsymbol{\omega}]$, do not coincide with $|\boldsymbol{\Omega\omega}\rangle$ because they do not satisfy (3c). The LWS $|\boldsymbol{\Omega\omega}\rangle$ can, however, be constructed from $|\boldsymbol{\Omega}\rangle$ by the raising operator technique (Bincer 1977, 1978a, b, 1982). By definition, the raising operators R_i, $i = 1, \ldots, N$, for osp$(1/2N, \mathbb{R}) \supset$ sp$(2N, \mathbb{R})$, are operators acting in the carrier space of $(\boldsymbol{\Omega}\rangle$, and transforming any LWS $|\boldsymbol{\Omega\omega}\rangle$ into another LWS $|\boldsymbol{\Omega\omega}^i\rangle$, where $\omega_j^i = \omega_j + \delta_{ji}$:

$$R_i|\boldsymbol{\Omega\omega}\rangle = N_i(\boldsymbol{\omega}, \boldsymbol{\omega}^i)|\boldsymbol{\Omega\omega}^i\rangle. \tag{12}$$

Here $N_i(\boldsymbol{\omega}, \boldsymbol{\omega}^i)$ is some normalisation coefficient. Equation (12) is equivalent to the conditions

$$\begin{aligned} &[E_{jj}, R_i] = \delta_{N+1-j,i}R_i \qquad [E_{jk}, R_i]|\boldsymbol{\Omega\omega}\rangle = 0 \qquad j > k \\ &[D_{jk}, R_i]|\boldsymbol{\Omega\omega}\rangle = 0. \end{aligned} \tag{13}$$

To solve (13), it is convenient to denote the osp$(1/2N, \mathbb{R})$ generators by $\Lambda_{\alpha\beta}$, V_α, where greek indices run over $\pm 1, \ldots, \pm N$, and

$$\begin{aligned} &\Lambda_{ij} = D^{\dagger}_{ij} \qquad \Lambda_{-i,-j} = D_{ij} \qquad \Lambda_{i,-j} = \Lambda_{-j,i} = E_{ij} \\ &V_i = F^{\dagger}_i \qquad V_{-i} = F_i \qquad i, j = 1, \ldots, N. \end{aligned} \tag{14}$$

It is then obvious that the raising operators assume a form similar to that of the raising operators for wsp$(2N, \mathbb{R}) \supset$ sp$(2N, \mathbb{R})$ (Quesne 1988). In both cases, they indeed only depend on the vectorial character of V_α. Hence, R_i can be expressed as

$$R_i = -[V \cdot T^i]_{N+1-i} = -\sum_{\alpha\beta} g^{\alpha\beta} V_\alpha T^i_{\beta, N+1-i} \tag{15}$$

where $g^{\alpha\beta}$ is the inverse of the sp$(2N, \mathbb{R})$ metric tensor $g_{\alpha\beta} = (\alpha/|\alpha|)\delta_{\alpha,-\beta}$, T^i is given by

$$\begin{aligned} &T^i = \prod_{j=1}^{N} (\Lambda - \sigma_j g) \prod_{k=N+2-i}^{N} (\Lambda - \tau_k g) \\ &\sigma_j = \omega_{N+1-j} + j - 1 \qquad \tau_k = 2N + 1 - k - \omega_{N+1-k} \end{aligned} \tag{16}$$

and $\Lambda \cdot \Lambda$ is to be understood as the tensor operator whose components are

$$(\Lambda \cdot \Lambda)_{\alpha\beta} = \sum_{\gamma\delta} g^{\gamma\delta} \Lambda_{\alpha\gamma} \Lambda_{\delta\beta}. \tag{17}$$

The normalisation coefficient $N_i(\boldsymbol{\omega}, \boldsymbol{\omega}^i)$ in (12) is, however, different from that obtained for $\mathrm{wsp}(2N, \mathbb{R}) \supset \mathrm{sp}(2N, \mathbb{R})$, because explicit use of the anticommutation or commutation properties of V_α (depending on which ones do apply) is made in its calculation. For $\mathrm{osp}(1/2N, \mathbb{R}) \supset \mathrm{sp}(2N, \mathbb{R})$, the result is

$$\begin{aligned} N_i(\boldsymbol{\omega}, \boldsymbol{\omega}^i) = (-1)^m 2(\Omega_i - i) & \left(\prod_{j=1}^{m} (\Omega_{p_j} - \Omega_i + i - p_j + 1) \right) \left(\prod_{j=1}^{N-l-1} (\Omega_i + \Omega_{q_j} - i - q_j) \right) \\ & \times \Bigg[(\Omega_i - i + 1) \left(\prod_{j=1}^{l} (\Omega_i + \Omega_{p_j} - i - p_j + 1)(\Omega_i + \Omega_{p_j} - i - p_j + 2) \right) \\ & \times \left(\prod_{j=m+1}^{l} (\Omega_i - \Omega_{p_j} + p_j - i - 1)(\Omega_i - \Omega_{p_j} + p_j - i)^{-1} \right) \\ & \times \left(\prod_{j=1}^{r} (\Omega_{q_j} - \Omega_i + i - q_j)(\Omega_{q_j} - \Omega_i + i - q_j - 1) \right) \Bigg]^{1/2} \end{aligned} \tag{18}$$

where

$$\begin{aligned} & \omega_j = \Omega_j + \delta_{j,p_1} + \ldots + \delta_{j,p_l} \\ & \{p_1 \ldots p_m i p_{m+1} \ldots p_l\} \cup \{q_1 \ldots q_{N-l-1}\} = \{1 \ldots N\} \\ & p_1 < p_2 < \ldots < p_l \qquad q_1 < q_2 < \ldots < q_{N-l-1} \\ & p_m < i < p_{m+1} \qquad q_r < i < q_{r+1}. \end{aligned}$$

During the calculation of $N_i(\boldsymbol{\omega}, \boldsymbol{\omega}^i)$, the matrix elements of the odd generators $F_k^\dagger$ between two $\mathrm{sp}(2N, \mathbb{R})$ LWS are obtained in the form

$$\begin{aligned} \langle \boldsymbol{\Omega}\boldsymbol{\omega}^i | F_k^\dagger | \boldsymbol{\Omega}\boldsymbol{\omega} \rangle & \\ = \delta_{k,N+1-i} (-1)^{m+i-1} & \Bigg[(\Omega_i - i + 1) \\ & \times \left(\prod_{j=1}^{l} (\Omega_i + \Omega_{p_j} - i - p_j + 2)(\Omega_i + \Omega_{p_j} - i - p_j + 1)^{-1} \right) \\ & \times \left(\prod_{j=m+1}^{l} (\Omega_i - \Omega_{p_j} + p_j - i - 1)(\Omega_i - \Omega_{p_j} + p_j - i)^{-1} \right) \\ & \times \left(\prod_{j=1}^{r} (\Omega_{q_j} - \Omega_i + i - q_j - 1)(\Omega_{q_j} - \Omega_i + i - q_j)^{-1} \right) \Bigg]^{1/2}. \end{aligned} \tag{19}$$

As an illustration of the general results, let us consider the case of $\mathrm{osp}(1/4, \mathbb{R}) \supset \mathrm{sp}(4, \mathbb{R})$. The two raising operators R_1 and R_2, as defined in (15), depend on the $\mathrm{sp}(4,\mathbb{R})$ weight $\omega_1\omega_2$ that is being raised. Equivalent operators, independent of the weight, can, however, be recovered by taking (3) into account, and are given by (Quesne 1987):

$$\begin{aligned} R_1 &= 2F_2^\dagger(E_{22} - 1)(E_{11} + E_{22} - 3) - [2D_{12}^\dagger(E_{22} - 1) - D_{22}^\dagger E_{12}]F_1 - D_{22}^\dagger(E_{11} + E_{22} - 2)F_2 \\ R_2 &= 2[F_1^\dagger(E_{11} - E_{22}) + F_2^\dagger E_{12}](E_{11} - 2)(E_{11} + E_{22} - 3) \\ & \quad -[D_{11}^\dagger(E_{11} - E_{22} + 1)(E_{11} + E_{22} - 2) + 2D_{12}^\dagger E_{12}(E_{22} - 2) - D_{22}^\dagger E_{12}^2]F_1 \\ & \quad -2[D_{12}^\dagger(E_{11} - E_{22}) + D_{22}^\dagger E_{12}](E_{11} - 2)F_2. \end{aligned} \tag{20}$$

A generic osp(1/4, ℝ) irrep $\langle\Omega_1\Omega_2\rangle$ contains four sp(4, ℝ) LWS corresponding to $\langle\omega_1\omega_2\rangle = \langle\Omega_1\Omega_2\rangle$, $\langle\Omega_1+1, \Omega_2\rangle$, $\langle\Omega_1, \Omega_2+1\rangle$, and $\langle\Omega_1+1, \Omega_2+1\rangle$ respectively. The explicit form of the latter three directly results from (18) and (20) and is given by

$$
\begin{aligned}
&|\Omega_1\Omega_2, \Omega_1+1\Omega_2\rangle = \Omega_1^{-1/2} F_2^\dagger|\Omega_1\Omega_2\rangle \\
&|\Omega_1\Omega_2, \Omega_1\Omega_2+1\rangle \\
&\qquad = [(\Omega_2-1)(\Omega_1-\Omega_2)(\Omega_1-\Omega_2+1)]^{-1/2}[-(\Omega_1-\Omega_2)F_1^\dagger + F_2^\dagger E_{12}]|\Omega_1\Omega_2\rangle \\
&|\Omega_1\Omega_2, \Omega_1+1\ \Omega_2+1\rangle \\
&\qquad = \tfrac{1}{2}[\Omega_1(\Omega_2-1)(\Omega_1+\Omega_2-1)(\Omega_1+\Omega_2-2)]^{-1/2}[2(\Omega_1+\Omega_2-2)F_1^\dagger F_2^\dagger \\
&\qquad\quad -2(\Omega_1-1)D_{12}^\dagger + D_{22}^\dagger E_{12}]|\Omega_1\Omega_2\rangle
\end{aligned}
\tag{21}
$$

where we may also write

$$E_{12}|\Omega_1\Omega_2\rangle = (\Omega_1-\Omega_2)^{1/2}|\Omega_1\Omega_2(\Omega_2+1)\rangle \tag{22}$$

where (Ω_2+1) is a Gel'fand pattern of $[\Omega_1\Omega_2]$.

For the raising operators of osp(1/6, ℝ) ⊃ sp(6, ℝ), detailed expressions are also available (Quesne 1988).

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